Estimation of multifractal properties of random walks observed through finite probing windows

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Abstract

Random walks have fractal properties in the sense that a rescaled random walk has statistical properties very similar to that of the original, and appear qualitatively the same on any time scale. Many random walks have a mean square displacement that grows linearly in time. More generallly, the $2n$-th moment of the displacement scales as a power of time $t^{2nH(n)}$, where the scaling exponent H is known as the Hurst exponent. Monofractal walks have unique exponents $H(n) = H$, whereas multifractal walks have nonunique $H(n)$. There are a number of biological instances in which finite sized probing windows need to be considered, from examples of cells and bacteria in confined spaces to rodent populations of interest in epidemic spread. A monofractal random walk seen through a finite probing window will incorrectly appear to have multifractal properties [1]. We propose a method to correct for such spurious detection of multifractality. We then study the case in which the roaming region of the walker is itself of limited extent, so that a nonlinear interplay occurs between the roaming area and the window size.

1. L. Giuggioli, G.M. Viswanathan, V.M. Kenkre, R.R. Parmenter, and T.L. Yates: Effects of Finite Probing Windows on the Interpretation of the Multifractal Property of Random Walks, Europhysics Letters 77, 40004-p. 1–5 (2007).

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